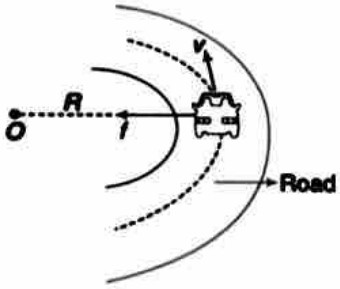
4.2 Circular Turnings

When a car moves on a level road, its weight is balanced by the normal reaction of the road. Any acceleration of the car is the result of friction force acting on it. When driver presses the accelerator or brakes or rotates the steering wheel, friction force on wheels assumes a direction so as to prevent slipping of wheels on the road. You will have a slightly better understanding of this after studying about *rolling motion* in second volume of this book.

Right now, it is enough to understand that it is friction which produces acceleration in a car moving on a level road.

Consider a vehicle moving along a curved road, which is in shape of a circular arc of radius R. The road is perfectly horizontal. The figure shows the top view of the situation. Vertical forces – normal reaction and weight – balance out.



The vehicle has a tendency to skid outwards. To prevent this, the friction force takes a radially inward direction. We are assuming that the vehicle is maintaining a constant speed so that there is no component of friction in tangential direction.

$$f = \frac{mv^2}{R}$$

If v is high, friction is also high. But friction cannot increase beyond a limit.

$$f \le \mu N = \mu mg$$

$$\Rightarrow \quad \frac{mv^2}{R} \le \mu mg$$

$$\Rightarrow \quad v \le \sqrt{\mu Rg}$$

For safely negotiating the curve, the speed should not exceed $v_o = \sqrt{\mu Rg}$. Beyond this speed, friction will fail to provide the necessary centripetal force and the vehicle will skid out of the road.

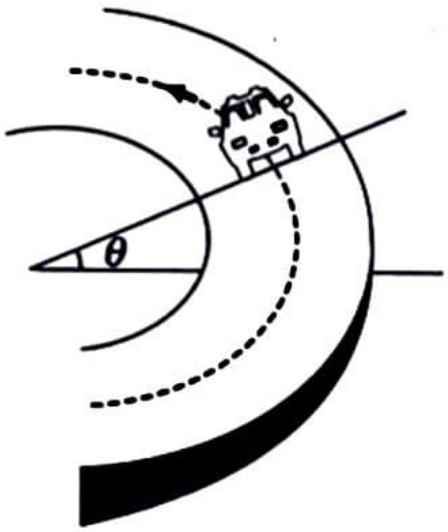
If the turning is sharp (i.e., R is small), the maximum speed limit for safe turning becomes small. A speeding vehicle is likely to skid. If it rains, coefficient of friction becomes smaller and the situation gets even worse. Relying on friction can prove to be dangerous in such situations.

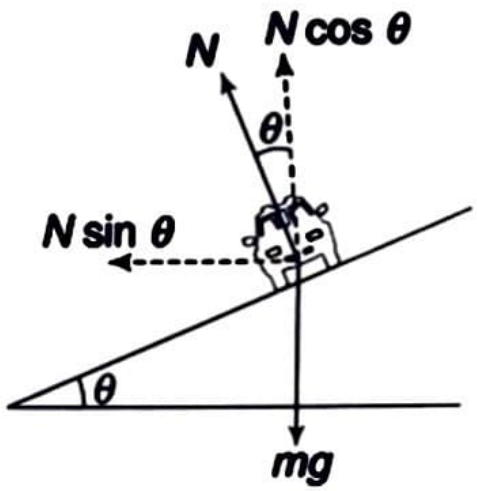
A simple but effective way to minimise the risk of skidding is by banking the roads.

4.2.1 Banking of roads

Roads are banked at turnings so that the outer boundary of the road is elevated as compared to the inner one. The sloping angle (shown as θ in the figure) is known as banking angle. The second figure shows the vertical section of the road and the vehicle is going into the *Plane of the figure at speed* v. Let's define correct speed (v_o) as the speed at which horizontal component of the normal force (N) is just sufficient to provide the necessary centripetal force. For vehicle travelling at correct speed friction will adjust itself to zero.

For a car travelling at correct speed, we have





$$N\cos\theta = mg \qquad \ldots(1)$$

$$N\sin\theta = \frac{mv_0^2}{R} \qquad \ldots(2)$$

where R is the radius of curvature of the turning. Dividing (2) by (1) gives

$$\tan\theta = \frac{v_0^2}{Rg}$$

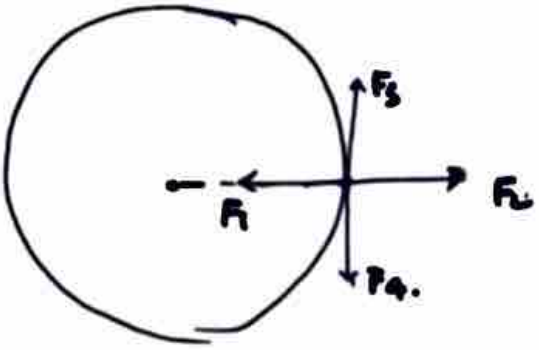
$$\Rightarrow \qquad v_o = \sqrt{Rg\tan\theta}$$

Usually, R is decided by the space available for turning (e.g., on hills, R will be small). A civil engineer designs the road for an expected speed by properly selecting a banking angle (θ).

In practice, a vehicle may negotiate a curve at a speed which is slightly less or more than the correct speed. In such cases, friction helps and adjusts the horizontal force to correct the value of required centripetal force.

CIRCULAR DYNAMICS.

If a particle is moving in a circle, resolve all forces along radius & perpendicular to radius. $|\vec{F_1}| > |\vec{F_2}|$.



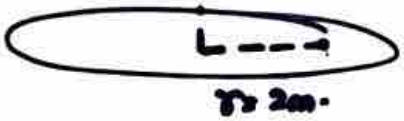
① $\vec{F_1} - \vec{F_2}$ = Centripetal force (F_c) $= \frac{mv^2}{r}$ or $m\omega^2 r$.

② $\vec{F_3} - \vec{F_4} = mra$ — (not compulsory). tangential force (F_t).
$= m \cdot \frac{dv}{dt}$

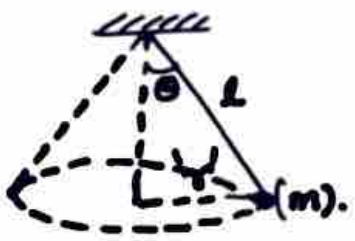
* for uniform circular motion $F_t = 0$.

Eg. A table is in the a circular shape as shown. A block of mass 5kg is kept at a distance 2m from centre. If the table is rotating with $\omega = 10$ rad/s. The limit required so that the block does not slide is?

$\omega = 10$ rad/s



Eg. CONICAL PENDULUM.

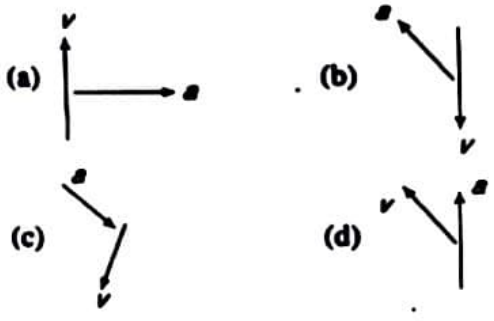


Find the velocity of the bob which is rotation as shown. (uniform circular motion).

1. A particle is moving in a circle with a constant speed. Which of the followings is true regarding the particle
   (a) It is acted upon by a constant force and its acceleration remains constant.
   (b) It is acted upon by a constant force but its acceleration is variable.
   (c) The force experienced by the particle as well as its acceleration-both are variable.
   (d) Newton's laws of motion are not applicable in case of circular motion.

2. You are sitting in a car which negotiates a curve of radius 20 m with a uniform speed of 20 ms^{-1}. Your mass is 60 kg and you are sitting upright on the seat with no side support. The horizontal component of force applied by the seat on your body will be
   (a) 1,000 N (b) 1,500 N
   (c) 1,800 N (d) 1,200 N

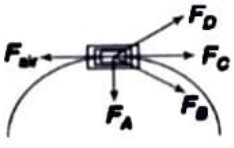
3. Shown here are the velocity and acceleration vectors for an object in several different types of motion. In which case is the object slowing down and turning to its right?



4. A block of mass m is projected on a smooth horizontal circular track with a velocity v. What is the average normal force (define it as mass multiplied with average acceleration) exerted by the circular walls on the block during its motion from A to B.

   (a) $\frac{mv^2}{R}$ (b) $\frac{mv^2}{\pi R}$
   (c) $\frac{2mv^2}{R}$ (d) $\frac{2mv^2}{\pi R}$

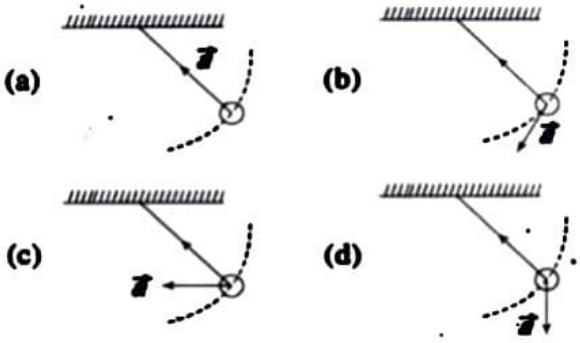
5. A car travels with a constant speed on a circular road on a level ground. In the diagram, F_{air} is the force of air resistance on the car. Which of the other forces shown best represents the horizontal force of the road on the car's tires?



   (a) F_A (b) F_B
   (c) F_C (d) F_D

6. An unbanked circular highway curve on level ground makes a turn of 90°. The highway carries traffic at 108 kmh^{-1}, and the centripetal force on a vehicle is not to exceed $\frac{1}{10}$ of its weight. What is the approximate minimum length of the curve, in km?
   (a) 1.4 km (b) 1 km
   (c) 0.6 km (d) None of these

7. A simple pendulum is oscillating in a vertical plane. For a position when the displacement of the bob is less than maximum, the acceleration vector $\vec{a}$ of the bob may be correctly represented in which of the following diagrams.



8. Keeping the angle of banking of the road constant, the correct speed of the vehicles plying on a road is to be increased by 5 per cent. The radius of curvature of the road will have to be changed from 20 m to _____.
   (a) 22 m (b) 40 m
   (c) 24.2 m (d) 14.4 m

9. Water in a bucket is whirled in a vertical circle with a string attached to it. Dimension of the bucket is negligible with respect to the length (R) of the string and the bucket is being moved with a constant speed v. Water does not fall down even when the bucket is inverted at the top of its path. We conclude that-
   (a) $mg = \frac{mv^2}{R}$ (b) $mg > \frac{mv^2}{R}$
   (c) $mg < \frac{mv^2}{R}$ (d) None of these

10. A motorcycle is going on an over bridge of radius R. The driver maintains a constant speed. As the motorcycle is ascending on the over bridge, the normal force on it would _____.