JMR'S Physics
Premier Top Notch Coaching Institute in PHYSICS for
JEE (IIT's and Other Engineering Ent., BITSAT) AIIMS, NEET (Medical Ent.)

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Discussion ① Elasticity.

① Note: when any object falls freely, there are no restoring forces at an section

$mg - F = ma$
$= mg$
$F = 0$

∴ No stresses are developed in the body
∴ Strain of any section = zero.

Problem ① a)

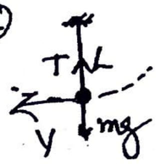
Y = Young's modulus of elasticity
mass = m
length = l
radius = r

a) The rod shown falls freely, then stress at middle of rod

1) $\frac{mg}{2\pi r^2}$ 2) $\frac{mg}{4\pi r^2}$ 3) $\frac{mg}{\pi r^2}$ 4) zero.

b) In the above problem what is the elongation of rod Ans: zero.

Note ②

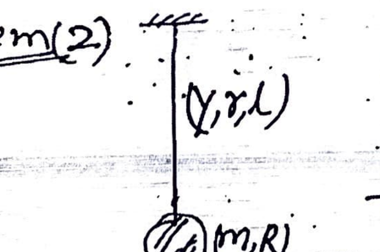


$T - mg = \frac{mv^2}{L}$ ①. Elongation

$e = \frac{Fl}{AY}$

Area of C.S

Problem (2)



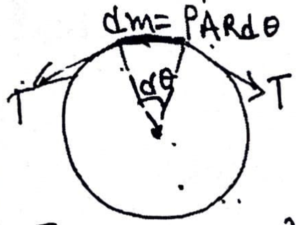
When the shown Pendulum oscillates the bob grazes surface. Find the speed of bob?

Div. ②

③ Note: For a Ring of Radius 'R' made of material of density 'ρ' and breaking stress 'σ' rotates about it's axis. Then the maximum angular velocity 'ω_{Max}' without rupture occuring

$$\boxed{\omega_{Max} = \frac{1}{R}\sqrt{\frac{\sigma}{\rho}}}$$

$dm = \rho A R d\theta$



$T d\theta = \rho A R d\theta \cdot \omega^2 R$

$\sigma A = \rho A \omega^2 R^2$

$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{\sigma}{\rho}}$

Problem(3) There are two Rings of same radius, whose breaking stresses are in the ratio 1:2 and densities in the ratio 3:4. If ~~Maximum~~ minimum time period of revolution of first ring without rupturing is T_0 find it's value for second Ring?

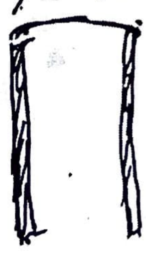
④ Note(4)(i) The Maximum internal pressure that can be sustained by a spherical shell of radius R, thickness t, and breaking stress σ is

$P \times \pi R^2 \le \sigma\, 2\pi R t$

$P_{Max} = \frac{2\sigma t}{R}$

$$\boxed{P_{Max} = \frac{2\sigma t}{R}}$$

For a cylinderical shell



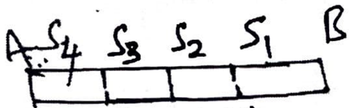
$P \times 2R \times l$
$\le 2l \times t\,\sigma$

$P_{Max} = \frac{\sigma t}{R}$

$P_{Max} = \frac{\sigma t}{R}$

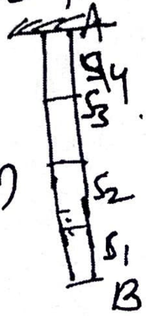
Prob 4). Two spherical shells of same material, have their radii in the ratio 1:2 and thickness in the ratio 2:1. Find the ratio of maximum pressures they can sustain?

Note 5)



Uniform rod AB

Suspended from ceiling as shown

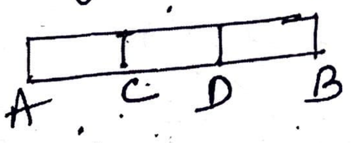


$m = \rho \times AL$

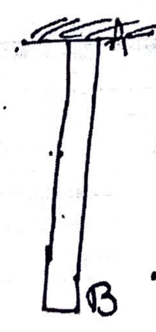
total elongation $= \frac{mgL}{2AY}$.

Elongations of previously equal sections $S_1, S_2, S_3, S_4, \ldots$ are in the ratio $1 : 3 : 5 : 7 : \ldots$

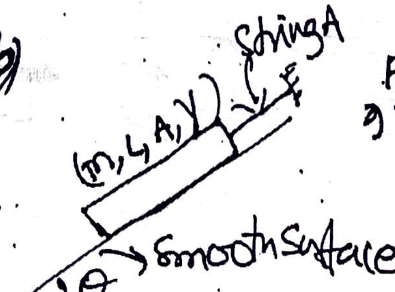
Prob 5a). A uniform rod AB of density 'ρ', length 'L' and of young's modulus of elasticity 'Y', has C and D as trisection points.



If rod is suspended from ceiling as shown, Find elongations of portions AC, CD, and DB?



b)



Find elongation of rod shown.
(i) If string A is present
(ii) if string A is cut?

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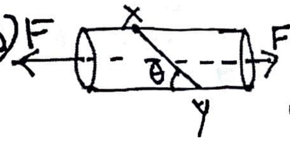
Discussion (4)

Note(6)



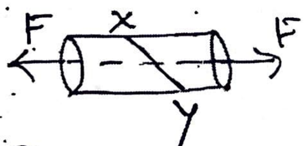
If Area of cross section of rod is 'A'
Area of section XY is $A/\sin\theta$.

on section XY (i) Normal stress $= \dfrac{F\sin^2\theta}{A}$

(ii) tangential (or) Shear stress $= \dfrac{F}{A}\sin\theta\cos\theta$.

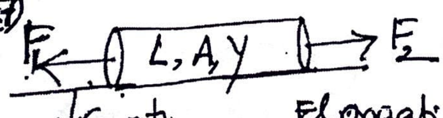
for tangential stress maximum $\theta = 45^\circ$

Problem(6) Forces are applied on a cylinderical rod of radius 'R' as shown. Let there be a section XY. What can be the maximum shear stress on section XY and at that instant what is normal stress on it?



Note(7)

L → length, A → Area, Y = Young's modulus.

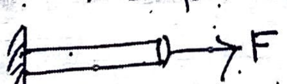


Elongation of rod $= \dfrac{(F_1+F_2)\,L}{2\,AY}$

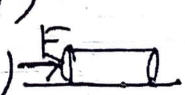
(tensile force to be taken +ve
compressive force " " -ve)

e +ve elongation; -ve compression.

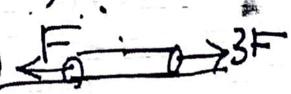
Problem(7) Find elongation or compression in the following? Each rod (L, A, Y)

(i) 

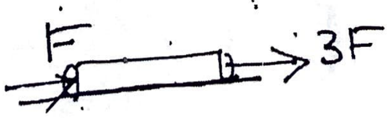
(ii) 

(iii) 

(iv) 

(v) 

(vi) 

(vii) 

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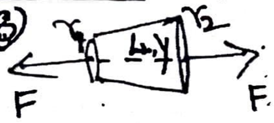
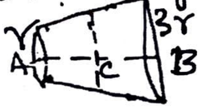
l Discussion ⑤.

Note(8)

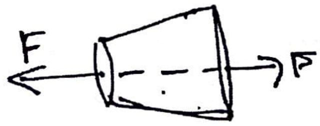


elongation $= \dfrac{Fl}{\pi r_1 r_2 Y}$

Problem(8) uniform tapered rod (AB) is as shown (r, $3r$ are end radii)

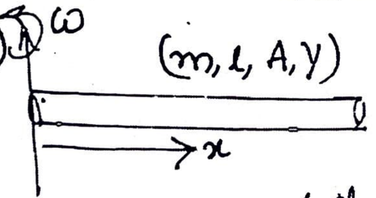


'C' is middle section.
If forces are applied as shown.



find ratio of elongations of AC and CB?

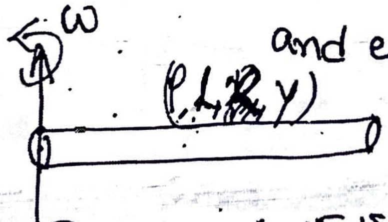
Note(9)



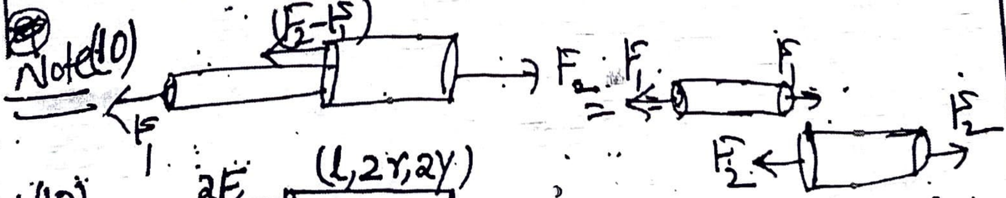
(Tension)
$$T_{(x)} = \frac{m\omega^2}{2l}(l^2 - x^2)$$

& elongation
$$e = \frac{m\omega^2 l^2}{3AY} = \frac{\rho\omega^2 l^3}{3Y}$$

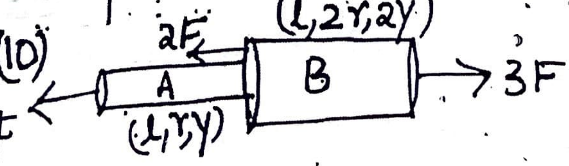
Prob(9) A uniform rod of radius R, length L, ~~and~~ density ρ, young's modulus Y is rotating with angular velocity 'ω' as shown. Find tension at middle of rod and elongation of rod



Note(10)



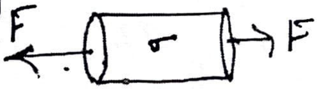
Prob(10)



Find ratio of elongations of rods A and B?

Discussion (6)

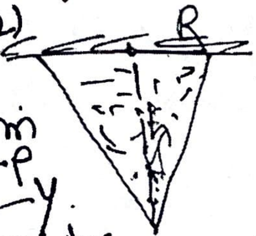
Question: what is meant by (Poisson's ratio) $\sigma = -0.5$

Note(11) F  F

volume strain = Longitudinal strain $(1-2\sigma)$

Problem(11) When a cylinderical rod is subjected to tensile forces, there is no change in volume. Find σ?

Note(12)

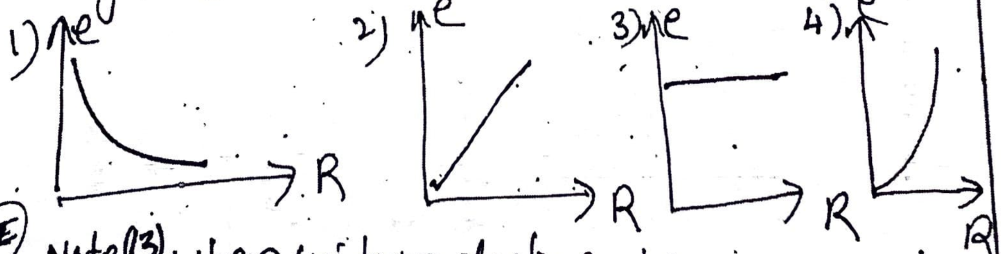


mass ← m
density ← ρ
Y
young's modulus

Conical Rod suspended as shown. Elongation Due to self weight

$$e = \frac{mgh}{2\pi R^2 Y} = \frac{\rho g h^2}{6Y}.$$

Prob(12) Conical rods of given material are suspended from roof. The plot of variation of elongation due to self weight and their radii is



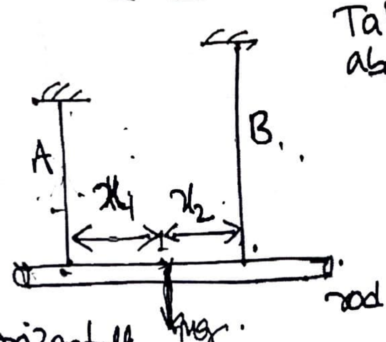
Note(13) when uniform elastic Rod is suspended from Roof

Elastic potential energy stored in it is $\frac{m^2g^2 l}{6AY} = \frac{\rho^2 g^2 A l^3}{6Y}$

Prob(13) Two uniform rods of same material and length have their radii in the ratio 1:2. If they are suspended from roof find Ratio of elastic potential energies in them?

Dissertation (7)

Note(14)



Taking moments about G of rod.

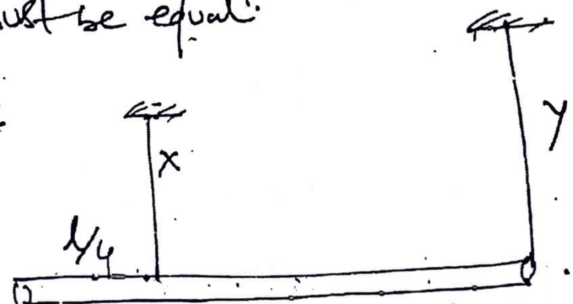
$$\boxed{\frac{F_A}{F_B} = \frac{x_2}{x_1}}$$

$F_A + F_B = mg$

Initially rod suspended horizontally.

For the rod to remain in horizontal position elongations in both wires A and B must be equal.

Prob(14)



uniform rod of length l.

In the fig. shown wire X has $\frac{1}{2}$ the length Y, double the radius of Y. Find the ratio of young's moduli of X and Y so that rod remains horizontal.)

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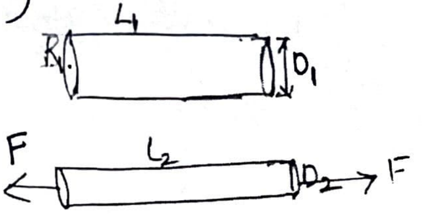
Elasticity – 2

1)



elongation $e = L_2 - L_1$

Longitudinal strain $= e/L_1$

lateral strain $= \dfrac{D_1 - D_2}{D_1} = \dfrac{R_1 - R_2}{R_1}$

Poisson's ratio $\sigma = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

$\sigma = \dfrac{\%\ \text{decrease in radius}}{\%\ \text{increase in length}}$

b) For a cylinderical rod subjected to tensile (or) compressive forces.

volume strain = longitudinal strain $\times (1 - 2\sigma)$

2) work done in stretching a wire (or) energy stored in a stretched wire.

$= \frac{1}{2} F e = \frac{1}{2}\, \text{stress} \times \text{strain} \times \text{volume}$

$\Rightarrow$ Energy stored / volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$

$= \frac{1}{2} \times \frac{(\text{stress})^2}{Y}$

$= \frac{1}{2} \times (\text{strain})^2 \times Y$

spring constant (or) stiffness of spring
↑
K

3) ~~~~ → F elongation $e = F/k$

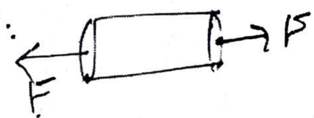
work done (or) potential energy stored

$= \frac{1}{2} k x^2 = \frac{1}{2} F x = \frac{1}{2} \frac{F^2}{k}$

Ex ① For a rod of length 10 cm and diameter 1 cm, poisson's ratio is 0.4. When subjected to tensile forces length becomes 10.2 cm. Find diameter?

Ex ⑮ from Redbook page 75..

Ex ②.

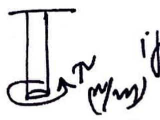


For the above rod, there is no change in volume, on forces application. Find poisson's Ratio?

3)a)

$F = mg$.

5)



if twisted through 'θ' radians

work done $= \frac{1}{2} \tau \theta$.

c) Rod

$=$ ~~~~ $k = AY/L$.

d) isothermal Bulk modulus of a gas $(K_{iso}) = P$ → pressure of gas

Adiabatic bulk modulus of gas

$(K_{Adia}) = \gamma P$

$\gamma = C_P / C_V$

as $\gamma > 1$

$K_{Adia} > K_{iso}$

compressibility $= \frac{1}{K} = \frac{1}{\text{Bulk modulus}}$

Page: 77: 34 to 45.

$\therefore$ *Bulk modulus* $K = \dfrac{F/A}{\Delta V/V} = \dfrac{FV}{A.\Delta V}$ ...(2)

For **gases and liquids,** the normal stress is caused by change in pressure *i.e.*

normal stress = change in pressure ΔP

$\therefore$ *Bulk modulus* $K = -\dfrac{\Delta P}{\Delta V/V}$ ...(3)

Negative sign indicates that the volume decreases if the pressure increases and vice-versa.

Equation (3) may be expressed as

$$K = -V\left\{\frac{\Delta P}{\Delta V}\right\} = -V\frac{dP}{dV}$$

Gases possess two types of volume elasticity

(i) *Isothermal elasticity,*

$$E_T = -V\left\{\frac{dP}{dV}\right\}_T = \text{Pressure, } P$$

(ii) *Adiabatic elasticity,*

$$E_s = -V\left\{\frac{dP}{dV}\right\}_S = \gamma P \text{ where } \gamma = \frac{C_P}{C_V}$$

The reciprocal of bulk modulus is called *compressibility*

i.e. Compressibility $= \dfrac{1}{\text{Bulk modulus}}$

3. *Modulus of rigidity,* η *:* It is defined as the ratio of tangential stress to shearing strain (*i.e.* strain of shape)

i.e. $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

If the lower face of a cube is fixed and tangential force is applied at the upper face of area A (as shown in fig.) the shape of cube changes and the strain is *shearing defined as the angular displacement (in radian) of any lateral section (like OA) along the direction of tangential force i.e.*

Shearing stress $= \theta = \dfrac{x}{L}$

$\therefore$ Modulus of rigidity $\eta = \dfrac{\text{Shearing stress}}{\text{Shear}} = \dfrac{F/A}{\theta}$

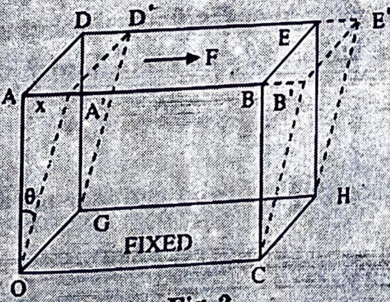


Fig. 2

4. *Poisson's Ratio* (σ) *:* When a force is applied along the length of wire, the wire elongates along the length but it contracts radially.

Then

Longitudinal strain $= \dfrac{\Delta L}{L}$

Lateral strain $= \dfrac{\Delta r}{r}$

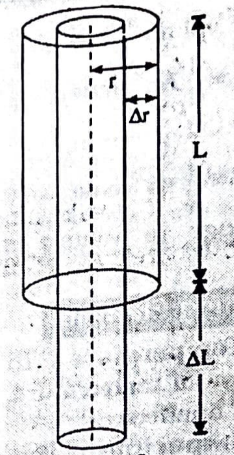


Fig. 3

The Poisson's ratio (σ) is defined as the ratio of lateral strain to longitudinal strain

i.e. Poisson's ratio $\sigma = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$= -\dfrac{\Delta r/r}{\Delta L/L}$

Negative sign indicates that change in length and radius r are of opposite sign *i.e.* if one decreases, the other increases.

The theoretical value of σ lies between -1 and 0·5, while the practical value of σ lies between 0 and 0·5.

The value of σ is 0·5, provided the change in volume is zero.

Fractional change in volume,

$$\frac{\Delta V}{V} = (1 - 2\sigma) \times \text{longitudinal strain}$$

4. Relations between Elastic Constants

There are four relations between elastic constants; Y, η, K and σ :

$$Y = 3K(1 - 2\sigma) \quad ...(1)$$

$$Y = 2\eta(1 + \sigma) \quad ...(2)$$

$$\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta} \text{ or } Y = \frac{9K\eta}{3K + \eta} \quad ...(3)$$

$$\sigma = \frac{3K - 2\eta}{6K + 2\eta} \quad ...(4)$$

5. Work done in deforming a body or Elastic Energy Stored

$$W = \frac{1}{2} \times \text{stretching force} \times \text{extention} = \frac{1}{2} F.\Delta l$$

As volume of wire $V = A \times L$, we have work done per unit volume of wire or elastic energy per unit volume of wire

$$\frac{W}{V} = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

i.e. **Resilience per unit volume**

$$= \frac{1}{2} \text{ stress} \times \text{strain} = \frac{1}{2} \times Y \times \text{strain}^2 = \frac{\text{stress}^2}{2Y}$$